Coalitional bargaining with private information*

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Abstract

In this paper, we consider a model of coalition formation in which one player has private information about her outside option. This player is also essential in that no coalition not including her can obtain any value. Values of coalitions depend on membership but not on the outside option, which only becomes relevant if someone leaves the bargaining. We show that, in any stationary equilibrium for high enough δ, the informed player never makes an informative or acceptable counter offer. If she rejects an offer from an uninformed player, the game ends. An uninformed player therefore calculates what offer maximizes his expected payoff given the amount he has to give other uninformed members of the coalition.

Keywords: asymmetric information, signalling, sequential bargaining

JEL Classification: C78, D82

1 Introduction

In this paper, we consider a model of coalition formation in which one player has private information. Whilst we shall discuss the relation with the existing literature later in the introduction, we point out now the following features of our model. We shall later discuss the main results.

There is one player, Player 1, who has private information about her outside option not available to other players. Using $v(.)$ to denote the characteristic function of the game, $v(\{1\})$ is privately known to Player 1. This differs from settings, such as many bilateral

*The authors would like to thank Jack Fanning, Brad Larsen and other participants at the 5th annual BEET conference, June 25-26, 2021, for valuable comments.

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bargaining games, where the private information affects the total surplus available to a non-trivial coalition. It also implies that if Player 1 foresees that future play might involve her taking her outside option, she should do so immediately.

We assume the Player 1 is an essential player, so that the value of any coalition S that does not include her is 0.

A player is not allowed to be a member of two or more non-identical coalitions, as is usual in this literature. Thus once Player 1 joins a coalition or takes her outside option, the game ends. This *one-coalition property* was a key feature in Selten [28] and Compte and Jehiel [10]. This implies that there is no incentive for any subset of players to wait for some other subset to leave before they form an alliance, a factor that causes possibly delayed agreement in extensive forms where the rejector of a proposer gets the initiative to make a new proposal (as in Chatterjee, Dutta, Ray, and Sengupta [5]).

Some other less crucial features of the model will be discussed later.

As an example of the setting we have in mind, consider a technology entrepreneur with an idea who is considering setting up her own firm. There are interesting questions of how much she has to disclose of her project in order for other players to write down a characteristic function but we do not address these here. The characteristic function values are considered common knowledge except for Player 1's outside option, which has a commonly known probability distribution.

Suppose Player 1 is negotiating with other players with different skills in order to set up this firm. Before she enters the negotiation, she has received an offer for her technology from an established firm. This buyout offer constitutes her outside option for the coalitional bargaining. The one-coalition property is natural in this setting, as is the fact that Player 1 is essential, since without Player 1's idea no other player can get a positive payoff.

We shall elaborate on this brief description of the environment later in this introduction. One additional requirement we place on the outside option is that it is significant, in that the lowest outside option is the maximum average coalitional worth of any coalition (treating Player 1 by herself as a coalition)-we call this the regularity condition. This is related to the "strategic bargaining" versus the Nash bargaining effect of the outside option (see Binmore, Rubinstein and Wolinsky [4]).

We model the negotiation through an extensive form used in Chatterjee, Dutta, Ray, and Sengupta [5], where a player makes a proposal consisting of a division of the coalitional worth among the members of the named coalition. The members of the coalition say yes or no in sequence. Player 1 is fixed to be the last person to respond. A player can reject the proposal in which case he gets to make a counter-offer or decide to quit the game. If a player quits, the proposal power goes to the next person in the sequence. Since Player 1 has to be a member of every coalition, she can make the offer if everyone else quits. If she quits too, the game is assumed to end with each player getting his individual worth (we shall assume everyone's individual worth is 0, except for Player 1).

For most of the paper, we assume that Player 1 is able to take her outside option only as a responder, not as a proposer. Since she has to be a member of every proposed coalition for that coalition to have value, this is not a significant restriction. However, for example, Chatterjee, Dutta, Ray, and Sengupta [5] allows a player to take his outside option only when proposing, so that one-person coalitions are treated on par with larger ones. Shaked [30] discusses these different assumptions and their effect on equilibrium play in a model of complete information bilateral bargaining. He asserts that these are best considered as representations of different institutional frameworks for bargaining.

The outside option for Player 1, denoted here as π, is assumed to be large enough to matter. The problem is thus rendered non-trivial because otherwise one could proceed by solving the complete information game and ignoring the outside option. (We are here adopting the outside option as something Player 1 chooses to take rather than something she is forced to take when the game ends exogenously, as explained in Binmore, Rubinstein and Wolinsky [4], otherwise an increase even in a small outside option would have an effect on the final outcome.)

Our results are derived mainly for sufficiently high values of δ. The equilibrium concept we use is stationary Perfect Bayes' equilibrium. Off-path beliefs do play a role but not a significant one. The definition of Perfect Bayes' equilibrium is adopted from Fudenberg and Tirole [15]; stationarity is used in many multiplayer bargaining papers and is usually justified on the basis of tractability and simplicity (a formal argument for simplicity of strategies for multiperson unanimity games is made by Chatterjee and Sabourian [6]).

Our main results are described informally in what follows. Perfect Bayes' equilibrium involves some assumption about off-path beliefs. These beliefs do not come into play in our most striking result, namely that (for high δ) any proposal by the informed player must be non-informative. We show that the other kind of equilibrium, where such a proposal is partially or fully informative, is not possible. Given the off-path belief, the equilibrium if an uninformed player makes an offer looks like this: either all responders named accept and the game ends or the informed player rejects and takes her outside option, thus also ending the game. If the informed player makes the offer, this denouement is postponed by one period. Thus the game ends within two periods. The example in Section 3 will make clear why this happens and the unravelling of beliefs that lead to the informed player not revealing any information. Of course, off the equilibrium path, the game could continue for a longer duration. The uninformed player has to trade off his payoff against those of the other uninformed players but also against the likelihood of getting nothing if the informed player rejects. This suggests the informed player, when she makes a rejected, uninformative proposal, will seek to transfer the proposal power to an uninformed player who will make the proposal with the highest probability of acceptance by the informed player (keeping other players' responses the same).

In terms of the usual questions asked about bargaining outcomes, our model displays a high degree of inefficiency and not in terms of delay. The inefficiency arises from the uninformed proposer underestimating the outside option of the informed player and thus precipitating the end of the game. This will not happen if the optimal proposal happens to be one where the informed party accepts with probability 1. If an efficient solution is reached, it is possible that the informed player gets more than she would have if her $v(\{i\})$ were commonly known.

Related literature. Some papers in extensive form models of characteristic function games have been mentioned already. Among the ones left out is Gul [17], which is very different in approach from this paper. Okada [23] studies a complete information game with proposers randomly chosen in each round and studies strictly superadditive games (under complete information). Perry and Reny [26] and Moldovanu and Winter [20] study games without discounting and identify equilibria that are in the core of the game, also under complete information.[1]

There are many fewer papers on coalition formation with some private information. Forges, Mertens and Vohra [13] take an ex ante mechanism design approach and define an ex ante core but do not deal with explicit bargaining protocols. Dutta and Vohra [11] define a different notion of core based on some consideration of blocking and information. There is no bargaining in these papers. The two papers we know of that involve non-cooperative modelling are both very different from ours and neither involves discounting- these are the papers by Serrano and Vohra [29] and Okada [24].

Serrano and Vohra [29] extend the notion of core to an incomplete information exchange economy by formalizing the coalitional decision to object (to a status quo allocation) via an intra-coalition simultaneous move single period Bayesian game. Okada [24] generalizes this idea of coalitional objection by allowing sequential one-stage intra-coalition unanimity voting, which allows for information transmission among members. Okada [24] further considers an alternating offer intra-coalition repeated bargaining game similar to ours, and formalizes objections that constitute stationary sequential equilibrium under the assumption that: proposals in this bargaining game are never informative.[2] In our paper, we find this property to be a necessary feature of any equilibrium.

The literature on bilateral bargaining might provide some more analogues to our work and we provide a selective and brief summary of this literature, omitting complete information papers. The most famous strand of this literature is that related to the Coase conjecture in which the seller (who is uninformed) makes offers and the buyer (who is

[1]There is also a large literature on complete information coalitional bargaining in settings with externalities across coalitions. For details see Ray [27].

[2]Okada [24] cites this exogenous restriction of non-informative offers as an application of the "*principle of inscrutability*" proposed by Myerson [21]. He also imposes another exogenous restriction, which presumes that agents respond using a type dependent cutoff rule. This property, too, is obtained as a necessary property for any equilibrium in our paper.

privately informed) accepts or rejects. If the lowest buyer value exceeds the seller value, a unique, weakly stationary equilibrium exists (Fudenberg, Levine and Tirole [14]). As offers become more frequent ($\delta \to 1$), the seller's offer converges to the lowest buyer value and the game ends almost immediately (Gul, Sonnenschein and Wilson [19])- this being known as the Coase conjecture. This is not particularly relevant to our results here since the informed party never makes offers. Gul and Sonnenschein [18] show that a similar result can hold if the informed party does make offers, under the condition that such offers, if not accepted, are uninformative. This condition is one that we prove as a characteristic of all stationary PBE in our model. Many variants seek to examine this conjecture (see Feinberg and Skrzypacz [12] and Cho [9] in different models. Of more relevance are the models with one-sided incomplete information where the informed party does make offers. Grossman and Perry [16] construct an equilibrium where the informed player's offers can be informative. Our result is similar, though the setting is very different, to Ausubel and Deneckere's "right to remain silent" (Ausubel and Deneckere [3]) where the informed player remains silent rather than give away any information. Admati and Perry [2] construct a model where the duration of the silence (the other player cannot interrupt) conveys information.

In two-sided incomplete information models such as Chatterjee and Samuelson [7] and [8], with each informed player being of two types, a player keeps making a non-informative offer as part of a randomised strategy until one of them reveals his type and the game becomes a one-sided incomplete information game.[3] The logic of the Coase conjecture then takes over, so that the player who is first to reveal loses all gains from trade as $\delta \to 1$ (Myerson [22] shows this last part in his textbook). Similar results are also presented in Abreu and Gul [1] and other papers built on their reputation model. Notice that in this literature, the war of attrition means the game continues, whilst in our model in the paper, the informed player quits because any attempt to use the information to do better fails in equilibrium.

2 Model

Consider an economic interaction involving players in $N = \{1, \dots, n\}$ described by an essential game where the following are common knowledge: (i) $v(N) = 1$, (ii) $v(S) = 0$ if $1 \notin S$, and (iii) $v(S) < v(N)$ for all $S \neq N$. However, the outside option of the essential player 1, that is, $v(\{1\}) := \pi$ is private knowledge, and is, publicly known to be distributed over $[\eta, 1], \eta \geq 0$ with a cumulative distribution function $F(.)$ that has positive density all over the support.

[3] Okada [25] focuses on a two agent alternating offers bargaining game with discounting where both agents have private verifiable types, and bargain over contracts. But he does not focus on general coalition formation like our analysis.

Define for all $S \subseteq N$, $\mathcal{L}_S$ to be the set of all possible linear orders on S. Further, for any $i \in N$, and any $\succ \in \mathcal{L}_{N \setminus \{i\}}$, define $\succ^i$ to be a linear order in $\mathcal{L}_N$ where all agents in $N \setminus \{i\}$ are ranked according to $\succ$, followed by i in the last position.

The agents bargain over forming a coalition using a bargaining game with '*sequential offers - rejector proposes protocol*'. So, at each information set, an *active agent* (that is, an agent who is yet to accept a proposal or quit in the game): either makes a proposal, or else responds to a proposal by either accepting it or rejecting it or else quitting the game altogether. A proposal by any active agent i is a tuple (S, y) where: (i) $i \in S$ with S being a subset of the set of all active agents containing at least two members, (ii) $y \in \mathbb{R}_+^{|S|}$, and (iii) $\sum_{j \in S} y_j \leq v(N)$. Whenever such a proposal (S, y) is made, members of S respond to this proposal sequentially according to any exogenous linear order $\succ_r^1$ where $\succ_r \in \mathcal{L}_{N \setminus \{1\}}$ - that is, where the informed party is ranked last.[4] If a proposal (S, y) is accepted, then the highest ranked active agent in $N \setminus S$ according to any exogenous linear order $\succ_p \in \mathcal{L}_N$; makes the next proposal. If (and only if) a proposal is rejected, all agents who have not yet quit the game, incur a period of delay, after which the rejector proposes.[5] Any agent who chooses to quit while responding to a proposal, realizes her outside option without any delay.

We make the following regularity assumption,

$$\mathcal{R}: \quad \eta \geq \max_{S \subseteq N} \frac{v(S)}{1 + (|S| - 1)\delta}.$$

This assumption ensures that the incomplete information problem is non-trivial at all information sets of our bargaining model. We require this restriction because the alternating offers nature of our bargaining protocol allows for possible information sets where dependence between beliefs and proposal decisions may break down. $\mathcal{R}$ rules out these possibilities as it binds outside options away from the stationary subgame perfect Nash equilibrium payoff in the complete information analogue of our bargaining game with zero outside options.[6] We discuss this point in greater detail in Section 4.

We use the equilibrium notion of Perfect Bayes' Equilibrium (PBE) for our bargaining game. A PBE assigns to each information set in the game, say I, an action and a belief.[7] This assigned action is the one that the agent with move at I, say k, is prescribed to undertake. The set of assigned beliefs are probability distributions on I, which in

[4] Relaxing this restriction would lead to non-existence of equilibrium.

[5] It is assumed to be common knowledge that; if a proposal is accepted, then it becomes a binding contract that is enforceable by courts, which in turn, implies that the proposer must make good on the promised payoff distribution. This assumption ensures that any proposal accepted on the equilibrium path must have a payoff distribution that sums up to the worth of the associated coalition.

[6] See Chatterjee, Dutta, Ray and Sengupta [5].

[7] In line with Fudenberg and Tirole [15], we assume that at all information sets, all uninformed parties hold the same belief about the informed party (see page 332).

our particular game of observable actions and perfect recall, translate into probability distributions over the Borel measurable subsets of $[\eta, 1]$. These beliefs must be consistent with Bayes' rule, wherever possible. Finally, the action assigned at I must be optimal for k, given her assigned beliefs at I.

In particular, we look for belief stationary PBE in pure strategies. These are PBE which satisfy the following property. At any two information sets I and I' such that the same agent, say i, has the move at both information sets: if a PBE assigns identical beliefs to i at both information sets, then the assigned action at both information sets must be identical too. Henceforth, in the paper, we refer to a belief stationary PBE as an '*equilibrium*'. We also define the following notation: for any $i \in S$, any $S \subseteq N$ and any $x \in \mathbb{R}^S_+$, $x_{-i} := (x_j)_{j \in S \setminus \{i\}}$.

3 Results

Let us denote the informed player with type $\pi \in [\eta, 1]$ as 1_π. We begin this section *by noting the fact* that for any equilibrium of this game, and any information set I on the equilibrium path, all uninformed parties share the same belief about the outside option of the informed party 1. This follows from the facts that: (i) past actions of 1 are equally observable across all uninformed agents, and (ii) the beliefs of uninformed parties must be formed on the equilibrium path in accordance to Bayes' rule.

And so, for any equilibrium, we define $G^i(I, B)$ for all $i \neq 1$, to be the continuation game which starts from the information set I where agent i has the move to propose, and all uninformed players have the same belief B about the distribution of private type π.[8] Similarly, for any equilibrium, define $G^{1_\pi}(I, B)$ as the continuation game starting from information set I where the informed player of type π has the move to propose, and all uninformed players haver the same belief B about the distribution of private type π. For simplicity of notation, we often: (1) suppress the argument I in the notation for a continuation game wherever the relevant information set is clear from the context, and (2) drop the subscript π (when the relevant outside option is clear from the context), and write this continuation game as $G^1(B)$.

3.1 An example

We begin by presenting the following example, which provides an informal and intuitive exposition of the nature of equilibria in our bargaining game. Consider a bargaining setting where: $v(N) = 1$, $v(12) = 0.9$, $v(13) = 0.45$, $v(1) := \pi \sim unif[0.4, 1]$, and $v(S) = 0$ for all other $S \subset N$. Fix $\delta = 0.8$, and suppose that 2 is the first proposer.

[8]It can easily be seen that B denotes a pair consisting of a measurable subset of $[\eta, 1]$, and a distribution over it.

Suppose that:

- 2 and 3 always propose formation of coalitions $\{1,2\}$ and $\{1,2,3\}$, respectively. The latter proposal is always accepted by 2. 1 always makes an uninformative proposal that is rejected by either of 2 or 3.

- 2 and 3 accept a proposal if and only if each uninformed member of the coalition to be formed, is offered an amount at least as great as δ times the maximum expected payoff that she can obtain by making a proposal herself. Any type 1_π accepts a proposal if it offers her at least π, or else she quits.

We argue below, informally, that when all agents are expected to play in a manner consistent with the description above, no agent can benefit by deviating unilaterally. To see this, note that proposing $\{1,2\}$ is better than proposing $\{1,2,3\}$ for agent 2. That is because, the expected payoff by proposing $\{1,2\}$ and offering any amount y_1 to 1 is $(0.9 - y_1)\text{Prob}(\pi \leq y_1) = \frac{(0.9-y_1)(y_1-0.4)}{0.6}$, which is maximized when $y_1 = 0.65$ (the corresponding probability of acceptance is $\frac{5}{12}$). Thus, the (maximum) expected payoff to 2 by proposing $\{1,2\}$ is 0.104. Arguing similarly, the expected payoff to 3 by proposing $\{1,2,3\}$ in a manner that is acceptable to 2 is, $\max_x (1-x)\text{Prob}(\pi \leq x) - 0.8 \times 0.104 = 0.0666$, with the maximizing x value being 0.7, and the corresponding probability of acceptance being 0.5. Now consider the maximum expected payoff that 2 can get by deviating and proposing $\{1,2,3\}$ in an acceptable manner. This value is given by $\max_x (1-x)\text{Prob}(\pi \leq x) - 0.8 \times 0.0666 = 0.0966$ which is less than 0.104, implying that proposing $\{1,2\}$ is more profitable for 2 than proposing $\{1,2,3\}$. Finally, 3 will not find it profitable to deviate and propose $\{1,3\}$ because the maximum available surplus to negotiate over is $v(\{1,3\}) - 0.4 = 0.05$, which is less than 0.0666.

Finally, consider the informed party 1. Note that 1's proposal can never be informative on the equilibrium path, because an informative proposal would divide the support into at least two subintervals. For example, suppose type $\pi \in [0.4, 0.7]$ is prescribed to make an acceptable proposal to agent 3 offering her 0.7 while type $\pi \in [0.7, 1]$ is prescribed to make an unacceptable proposal which will be rejected by either of agents 2 and 3. Then, the type $\pi \in [0.7, 1]$ would be offered 0.85 in the next period, implying that all types in $[0.4, 0.7]$ would deviate to mimic the types in $[0.7, 1]$.

Now, as we show later in Theorem 1, any non-informative offer to be made by the informed party must be an unacceptable offer. Note that such an offer must be addressed to agent 3 as she is the uninformed agent prescribed to propose the maximum amount 0.7 to agent 1 in the next period. Thus, if agent 1 is the first proposer, she makes an unacceptable proposal, and so, her equilibrium payoff is: 0.56 if her type lies in $[0.4, 0.7]$, or else $\delta\pi = 0.8\pi$ (obtained by quitting in the second period).

As we argue formally in the forthcoming theorems, these kinds of proposal and acceptance behaviours are the only kinds permissible in any equilibrium. Also note that the equilibrium expected payoffs, when 2 is the first proposer, are $(0.65, 0.104, 0)$; and the coalition $\{1, 2\}$ forms on the equilibrium path immediately with probability $\frac{5}{12}$, or else the informed party quits. Compare this to a complete information setting where π is known to be, say, 0.7, and all other information remains unchanged. As $\delta \to 1$, we can obtain the limiting equilibrium payoffs from Chatterjee, Dutta, Ray, and Sengupta [5] to be $(0.7, 0.2, 0.1)$. Notice that uninformed agents 2 and 3 do badly under incomplete information, and the equilibrium outcome, too, is inefficient.

3.2 Main results

We now present the formal results underlying the example above. First, we present the proposition below, which establishes that for any equilibrium of the game, there can be no information set where any type of the informed party makes an informative proposal (a proposal that leads to any updating of beliefs held by uninformed parties by Bayes' rule).

Theorem 1. *The informed party never makes an informative proposal on the equilibrium path.*

Proof: Suppose there exists an equilibrium σ such that there exists some type $1_{\tilde{\pi}}$ who is prescribed to make an informative offer (one that leads to updating of beliefs of uninformed parties) at some information set on the equilibrium path. Fix I to be the first such information set on the equilibrium path (that is, at all earlier information sets on the equilibrium path, the informed party made non-informative offers, if called on to make offers), and consider the continuation game $G^1(I, B)$. Note that, by construction, the belief B must have an interval support $J := [a, b] \subseteq [\eta, 1]$.

Now for each type 1_π with $\pi \in J$, define x_π to be the equilibrium payoff from playing according to σ in the continuation game $G^1([a, b])$ that starts from I. Let $\bar{J} \subseteq J$ be the types of the informed party who are prescribed to make an unacceptable proposal at I.[9] Now there are three possibilities: (i) $\bar{J} = \emptyset$, (ii) $\bar{J} = J$, and (iii) $\bar{J} \neq \emptyset, \bar{J} \neq J$. We consider each possibility in the following discussion as a different case.

Case (i). In this case, our supposition is that: (a) all types in $J := [a, b]$ are prescribed to make acceptable proposals, and (b) there exists at least one type $1_{\tilde{\pi}}$ whose acceptable proposal is informative.[10] We argue below that for all types $\pi \in J$, $x_\pi = K$ where K is a non-negative real constant. If not, then there exists at least a pair of types in J, with different equilibrium payoffs (and hence, different prescribed acceptable proposals)

[9] Given σ, an unacceptable proposal made by informed agent 1 is a tuple (S, y) such that there exists some $j \in S \setminus \{1\}$ such that σ prescribes j to not accept this proposal.

[10] Given σ, a proposal (S, y) is acceptable if it is not unacceptable.

implying that the type with lower equilibrium payoff has a profitable unilateral one-deviation in mimicking the other type's equilibrium action at I, which would contradict our σ being equilibrium.

Now the acceptable proposal, say P', prescribed to be made by $1_{\tilde{\pi}}$ at J is informative if and only if there exist disjoint subintervals J_l, J_h of J, where each type in J_h makes the acceptable proposal P' (that is, $\tilde{\pi} \in J_h$), and all types in J_l make some other acceptable proposal $Q' \neq P'$. By sequential rationality, there exist types $\pi'_h \in J_h$ and $\pi'_l \in J_l$ such that $x_{\pi'_h} \neq x_{\pi'_l}$ (since the intervals J_h and J_l are disjoint), which leads to a contradiction to the aforementioned conclusion that $x_\pi = K, \forall \pi \in J$.[11]

Case (ii). In this case, our supposition is that: (a) all types in $J := [a, b]$ are prescribed to make unacceptable proposals, and (b) there exists at least one type $1_{\tilde{\pi}}$ whose unacceptable proposal is informative. Suppose that $1_{\tilde{\pi}}$ is prescribed to propose some unacceptable proposal P. Note that any unacceptable proposal, essentially, makes a demand for a payoff that is to be rejected by at least one of the uninformed parties to whom this proposal is addressed.

Now, in a manner similar to the previous case (i): the unacceptable proposal P prescribed to be made by $1_{\tilde{\pi}}$ at I, is informative if and only if there exist disjoint subintervals J_l, J_h of J, where each type in J_h makes the unacceptable proposal P (that is, $\tilde{\pi} \in J_h$), and all types in J_l make some other unacceptable proposal $Q \neq P$. Therefore, by sequential rationality, there exist types $\pi'_h \in J_h$ and $\pi'_l \in J_l$ such that; both agents $1_{\pi'_h}$ and $1_{\pi'_l}$ realize their equilibrium payoffs by accepting an offer made by some uninformed party after observing their unacceptable proposals on the equilibrium path.[12] However, sequential rationality also implies that these realized equilibrium payoffs must be unequal, that is, $x_{\pi'_h} \neq x_{\pi'_l}$. Hence, as argued earlier, either of these two types has the profitable unilateral one-deviation to mimic the other on the equilibrium path. And so, once again, we get a contradiction to σ being an equilibrium.

Case (iii). In this case, our first supposition is that there exist a pair of types $1_{\pi'}, 1_{\pi''}$ such that: (i) $\pi' < \pi''$, (ii) $\pi' \in \bar{J}$, and (iii) $\pi'' \in J \setminus \bar{J}$. Now, if $x_{\pi''} > x_{\pi'}$, then the type $1_{\pi'}$ has a profitable unilateral one deviation to mimic type $1_{\pi''}$, and make the same acceptable proposal as the one prescribed by σ to $1_{\pi''}$. On the other hand, if $x_{\pi''} < x_{\pi'}$, then it must be that $1_{\pi'}$ does not quit on the equilibrium path (or else $x_{\pi'} = \delta\pi' < \delta\pi'' \leq x_{\pi''}$); and so, it follows that there exists a profitable unilateral deviation by type $1_{\pi''}$ where she mimics

[11] Therefore, two distinct offers on the equilibrium path cannot give identical equilibrium payoffs, and so, we get a contradiction.

[12] Suppose there exists a subinterval where all types realize their equilibrium payoffs by quitting. In that case, the equilibrium payoff to all uninformed parties in the continuation game following the unacceptable proposal made by 1 is zero. This contradicts the notion of equilibrium as at least one uninformed party can do better in expected sense, by offering any value in the interior of the belief subinterval. Therefore, π'_h and π'_l are well defined.

the type $1_{\pi'}$ (using the same argument as before). Therefore, we can infer that

$$(\mathbf{a})\ \delta\pi'' \leq x_{\pi''} = x_{\pi'}.$$

Now, by Bayes' rule, the unacceptable proposal of type $1_{\pi'}$ would reveal her outside option to be *less* than π'' (who is supposed to make an acceptable proposal by assumption). And so, at all future information sets on the equilibrium path generated by σ, $1_{\pi'}$ would be offered an amount less than that offered to π'' by any uninformed party, implying that $x_{\pi'} < \delta\pi''$. But then, **(a)** implies that $\delta\pi'' < \delta\pi''$, which is a contradiction.
Therefore, it must be that that for all $\pi' \in \bar{J}$ and all $\pi'' \in J \setminus \bar{J}$, $\pi' \geq \pi''$. This implies that both $\bar{J}$ and $J \setminus \bar{J}$ are sub-intervals of J.[13] That is, there exists $d \in J$, $d \notin \{a, b\}$ such that σ prescribes the informed party to make an acceptable proposal if her outside option $\pi \in [a, d]$, or else make an unacceptable offer. Now as argued earlier in Case (i), all types 1_π with $\pi \in [a, d]$ must get the same equilibrium payoff K', and (**b**) $K' \geq \delta d$. Now if there exists a $\hat{\pi} \in (d, b]$ such that $x_{\hat{\pi}} < K'$, then $1_{\hat{\pi}}$ has profitable unilateral one deviation to mimic any type $1_{\pi'}$ with $\pi' \in [a, d]$ and get the higher payoff K'. Therefore, σ is an equilibrium only if (**c**) $x_{\hat{\pi}} \geq K', \forall \hat{\pi} \in (d, b]$. Now, σ must ensure that there is no profitable unilateral one deviation available to any type $\pi \in [a, d]$, where she mimics a higher type $\hat{\pi} \in (d, b]$. This would be true only if σ prescribes all types in $(d, b]$ to - not only make an unacceptable proposal at information set J - but also quit the game at the consequent response node on the equilibrium path. That is, (**d**) $\forall \hat{\pi} \in (d, b]$, $x_{\hat{\pi}} = \delta\hat{\pi}$. Therefore, from **(b)** and **(c)**, it follows that for all $\hat{\pi} \in (d, b]$, $\delta\hat{\pi} \geq K'$ implying (in limit) that $d = \frac{K'}{\delta}$.
However, this implies that at the information set that arises on the equilibrium path with positive probability, after an uninformed party rejects an unacceptable proposal made at information set I, the beliefs of uninformed parties are updated in accordance with Bayes' rule where they now believe that $\pi \in (d, b]$. Therefore, from **(d)** it follows that: upon observing an unacceptable proposal on the equilibrium path, at least one uninformed party makes a proposal offering some $\zeta \leq d$ to the informed party, in response to which the informed party quits. However, this gives the uninformed party an equilibrium payoff 0, and so, she has a profitable unilateral one deviation of offering some $\zeta' > d$ upon observing an unacceptable proposal on the equilibrium path (and getting a positive expected payoff as informed types in the interval (d, ζ') would accept). Thus, we get a contradiction to σ being an equilibrium.

□

Theorem 1 above, implies that for any equilibrium of this game, the informed agent must make the same proposal irrespective of her outside option at any information set

[13] More precisely, the interval $J \setminus \bar{J}$ is on the left of interval $\bar{J}$ on the real line.

where she is called upon to make an offer. In other words, no matter what the outside option, the informed party always chooses a passive strategy of bargaining with an uninformed party *only while responding*. We show below that if the discount factor is not too low, then this non-informative proposal must be unacceptable to the uninformed parties.

Theorem 2. *If $\delta \geq \eta$, there exists no equilibrium in which the informed party makes an acceptable proposal.*

Proof: Suppose not. That is, suppose that there exists an equilibrium σ such that the informed party 1 makes an acceptable proposal and $\delta \geq \eta$. By Theorem 1, such a proposal must be non-informative. Further suppose that: (i) under σ, 1 makes such an acceptable proposal P, for formation of a coalition $1 \in S \subseteq N$, at an information set where the informed party has rejected a proposal by the uninformed party, and (ii) this acceptable proposal gives 1 a payoff of X. Note that by Theorem 1 all types must make the same acceptable proposal, and so, sequential rationality requires that $X \geq \delta$. Therefore, by sequential rationality and Bayes' rule, uninformed parties must hold a belief that $\pi \leq \delta X$ at the start of this continuation game. And so, in case the uninformed parties reject this proposal and make a counteroffer, they offer 1 an amount $y_1 \leq \delta X$. Further, note that if $y_1 < \delta X$, then by our supposition, all types of informed party would reject this counteroffer proposal, leading to a period of delay without any further updating of beliefs. Thus σ would constitute an equilibrium only if $y_1 = \delta X$. Therefore, the uninformed parties would expect this counteroffer to be accepted by player 1 for sure, and so, by the complete information bargaining logic developed in [5], each uninformed party can get at least $\frac{v(S)-\delta X}{1+(|S|-2)\delta}$. Therefore, the proposal P by 1 would be acceptable only if each uninformed party is offered at least $\frac{\delta[v(S)-\delta X]}{1+(|S|-2)\delta}$. Therefore,

$$X \leq v(S) - \frac{\delta(|S|-1)[v(S)-\delta X]}{1+(|S|-2)\delta},$$

which implies that $\delta \leq X \leq \frac{v(S)}{1+(|S|-1)\delta}$.[14] So if $\delta \geq \eta$, we get a contradiction to our regularity condition.
Finally, consider the only other remaining possibility that 1 is the first proposer in the game. Belief stationarity requires that prescription of σ to 1 at the initial information set, should be same as that at the information set analyzed above. And so, the result follows. □

In light of Theorem 2, henceforth, we assume $\delta \geq \eta$ for all later results. The following theorem builds upon Theorems 1 and 2, to establish that the informed party never makes a counteroffer on the equilibrium path.

[14]The inequality implies that $X\left[1 - \frac{\delta^2(|S|-1)}{1+(|S|-2)\delta}\right] \leq \frac{(1-\delta)v(S)}{1+(|S|-2)\delta} \Leftrightarrow X[(1-\delta)+(|S|-1)\delta(1-\delta)] \leq (1-\delta)v(S)$.

Theorem 3. *The informed party never makes a counteroffer on the equilibrium path.*

Proof: Fix any equilibrium σ, and suppose there exists a type of the informed party $1_{\pi'}$ who rejects an offer and makes a counteroffer on the equilibrium path. Let I be the earliest of such information sets; that is, at all earlier information sets on the equilibrium path, no type of the informed party rejects and makes a counteroffer. Therefore, by Theorem 1, the equilibrium belief prescribed by σ to I must have an interval support, say, must be $[a, b]$.[15] Further, for each type π, let x_π denote the equilibrium expected payoff to the informed party 1_π when all players play σ in the continuation game starting from the information set I. Finally, let Y be the sure amount that $1_{\pi'}$ would realise by accepting at I.

Therefore, $x_{\pi'} \geq \max\{Y, \pi'\}$, which, by Theorem 2, implies that $1_{\pi'}$ must accept a proposal to end the game on the equilibrium path in this continuation game. This implies that all types of informed parties 1_π with $\pi \leq \min\{b, x_{\pi'}\}$ must get the same equilibrium expected payoff $x_{\pi'}$ in this continuation game, or else they would have a profitable unilateral one-deviation of mimicking $1_{\pi'}$ to get a greater payoff or vice-versa. This, in turn, implies that all agents 1_π with $\pi \leq \min\{b, x_{\pi'}\}$ must, on the equilibrium path, take the same actions as $1_{\pi'}$, or else their types would get revealed, and by sequential rationality, they must not get the same equilibrium payoff $x_{\pi'}$.[16]

Note that if there exists any type of the informed party $1_{\hat{\pi}}$ with $\hat{\pi} \in (x_{\pi'}, b]$ who is also prescribed to accept a proposal on the equilibrium path in the continuation game starting from I; then $x_{\hat{\pi}} \geq \hat{\pi} > x_{\pi'}$, which implies that $1_{\pi'}$ has the profitable unilateral one-deviation to mimic $1_{\hat{\pi}}$ and get a higher payoff. Therefore, if σ is a PBE, it must be that all informed parties with types in $\pi \in (x_{\pi'}, b]$ must quit at I.

Thus, on the equilibrium path, if uninformed parties observe that game has not ended after the proposal made at information set I, then they update their beliefs to types being distributed in $[a, \min\{b, x_{\pi'}\}]$ by Bayes' rule. This means that at all the information sets on the equilibrium path subsequent to I, by sequential rationality, uninformed parties must not offer the informed party a sure amount greater than Y, implying that $x_{\pi'} \leq \delta Y < Y$.[17] Thus, we get a contradiction. □

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[15] Note that the equilibrium belief at I must have an interval support. That is because: if equilibrium belief at I does not have an interval support, then it must have a support, say J, which is a collection of disjoint intervals. And so, there must exist types π^*, π^{**} such that $\pi^* \neq \pi^{**}$, $\pi^* \in J$, and $\pi^{**} \notin J$. Therefore, at some previous information set $\hat{I}$ on the equilibrium path, σ must have prescribed 1_{π^*} to reject and make a counteroffer while prescribing $1_{\pi^{**}}$ to either accept or quit. But then I cannot be the earliest information set where some type of the informed party rejects and makes a counteroffer. This contradicts our supposition in the proof. In fact, we can infer from Theorem 1 that this interval support must contain all possible types, that is, be $[\eta, 1]$.

[16] Here our regularity assumption on η is essential, as it ensures that the amount offered by uninformed parties on the equilibrium path depends on their beliefs in a non-trivial manner.

[17] This would follow from maximization of expected payoff over a smaller interval with the same lower bound.

Corollary 1. *For any $\pi \in [\eta, 1]$, the agent 1_π accepts a proposal on the equilibrium path if and only if it offers her an amount greater than or equal to her outside option π.*[18]

Proof: By Theorem 3, on the equilibrium path, the informed party either accepts a proposal or else quits. Therefore, at any information set on the equilibrium path, if a type of the informed party is prescribed to quit, then her outside option must not be less than the sure amount she was offered, and if she is prescribed to accept then her outside option must not be greater than the sure amount she was offered. Hence, the result follows. $\square$

In the following part of the paper, we use Theorems 1 and 3 to characterize the equilibrium of the game. To do so, we first define the following class of strategy-belief pairs Σ^*.

Definition 1. *Let Σ^* be the set of strategy-belief pairs σ^* that:*

1. *for any $i \neq 1$ and at any continuation game $G^i(B)$, i proposes any $(S^*_{B,i}, y^*_{B,i})$ where*

$$y^*_{B,i_{-i}} = \left(\bar{w}_1^{B,i}, \left\{ \frac{\bar{a}_j^{B,j}}{F_B(\Omega_{B,\bar{w}_1^{B,i}})} \right\}_{j \in S^*_{B,i} \setminus \{1,i\}} \right) \quad \text{and} \quad \sum_{j \in S^*_{B,i}} y^*_{B,i_j} = v(S^*_{B,i})$$

such that $(S^*_{B,i}, \bar{w}_1^{B,i})$ solves

$$\max_{(T,y_1) \in \rho_i(N) \times [\eta,1]} \left\{ \frac{[v(T) - y_1] F_B(\Omega_{B,y_1})}{1 + \{|T| - 2\}\delta} \right\}$$

where $\rho_i(N) := \{S \subseteq N | i \in S\}$, $\Omega_{B,y_1} := \{\pi \in B : \pi \leq y_1\}$, and $F_B(.)$ is a distribution over $[\eta, 1]$ such that: (i) it is the same as the distribution associated with belief B over its support J, (ii) $F_B(x) = 0$ if $x \in [\eta, inf\ J]$, and (iii) $F_B(x) = 1$ if $x \in [sup\ J, 1]$.

Further, for all $j \in N$,

$$\bar{a}_j^{B,j} := \frac{\delta \left[v(S^*_{B,j}) - \bar{w}_1^{B,j} \right] F_B(\Omega_{B,\bar{w}_1^{B,j}})}{1 + \{|S^*_{B,j}| - 2\}\delta} .$$

At any response node where i's belief distribution is $F_B(.)$, she accepts a proposal offering any amount $x \in [\eta, 1]$ to the informed party; if and only if all uninformed agents j addressed in it, are offered a (sure) share at least as great as $\frac{\bar{a}_j^{B,j}}{F_B(\Omega_{B,x})}$.

[18] Ties may be broken in any arbitrary way. It is inconsequential as our prior belief is a density function.

2. *At any continuation game $G^1(B)$, the informed party, irrespective of her outside option, makes an unacceptable proposal (T_B^*, z_B^*) such that there exists a $j \in T_B^* \backslash \{1\}$ such that $z_{Bj}^* < \bar{a}_j^{B,j}$. At any response node, 1_π accepts a proposal if it offers her an amount at least as great as π or else she quits.*

3. *At any off the prescribed path (by σ^*) information set where Bayes' rule cannot be applied, if an uninformed $i \neq 1$ receives an acceptable proposal (as per the condition 2 above) from the informed party 1_π, then she changes her beliefs to put full probability on the event that $\pi = \eta$. At any other off the prescribed path information set where Bayes' rule is not applicable, any uninformed party keeps her beliefs unchanged as those held at the previous period. At every other information set, beliefs are updated in accordance with Bayes' rule.*

Remark 1. *It is easy to see that irrespective of the prior distibution of private information, $\Sigma^* \neq \emptyset$. Note that any strategy-belief pair $\sigma^* \in \Sigma^*$ leads to the different paths of play depending on the identity of the first proposer. That is, when all agents are playing as prescribed by σ^*: if an uninformed party $i \neq 1$ is the first proposer, then she proposes a coalition $S^*_{[\eta,1],i}$ and offers the informed party $\bar{w}_1^{[\eta,1],i}$ (that is, makes the proposal $(S^*_{[\eta,1],i}, y^*_{[\eta,1],i})$ as described in Definition 1 above) - which either leads to (i) an acceptance after which the game ends, or else (ii) quitting of the informed party after which the game ends. On the other hand, if the informed party is the first proposer when all agents are playing as prescribed by σ^*, then she makes an unacceptable proposal leading to a period of delay, after which the first rejector as per $\succ_r^1$ (say 2) proposes a $(S^*_{[\eta,1],2}, y^*_{[\eta,1],2})$, which in turn leads to, as before, either an acceptance or else quitting of the informed party.*

Now, we present the first main result of this section. It characterizes the equilibrium of our bargaining game when the underlying characteristic function is symmetric around the informed party 1.

Theorem 4. *For any $\succ_r^1, \succ_p$: whenever $|S| = |T| \implies v(S \cup \{1\}) = v(T \cup \{1\})$ for all $S, T \subseteq N \setminus \{1\}$,*

1. *any $\sigma^* \in \Sigma^*$ constitutes an equilibrium.*

2. *if there exists an equilibrium $\sigma_e \notin \Sigma^*$ then there exists a corresponding $\sigma_e^* \in \Sigma^*$ such that σ_e and σ_e^* prescribe the same actions and beliefs, at all information sets on the equilibrium path.*

Proof of (1): Consider any strategy $\sigma^* \in \Sigma^*$. Now fix any type π, any $i \neq 1$, any continuation game game $G^{1_\pi}(.)$, and consider any unilateral one-deviation by 1_π of making: either (a) an acceptable proposal or else (b) an unacceptable proposal different from the one prescribed in σ^*. Both possibilities would lead to an off the candidate equilibrium path (by σ^*) information set. By construction of σ^*, an acceptable proposal in the first case (a) would lead to a downward revision of the belief about type of 1 to the lowest possible value η. This implies that uninformed parties would never offer 1 any amount in excess of η on the equilibrium path, for all times in future. Thus, such a unilateral one deviation cannot be profitable. In the other case (b), σ^* implies that the beliefs of uninformed parties would remain unaffected, implying that such a one-deviation would bring no increment in expected payoff while incurring a period of delay. Hence, such a one-deviation, too, would not be profitable.

Finally, note that σ^* prescribes any uninformed party $i \neq 1$ to propose at any continuation game $G^i(B)$: a proposal $(S^*_{B,i}, y^*_{B,i})$ such that $y^*_{B,i_{-i}} = \left(\bar{w}_1^{B,i}, \left\{\frac{\bar{a}_j^{B,i}}{F_B(\Omega_{B,\bar{w}_1^{B,i}})}\right\}_{j \in S^*_{B,i} \setminus \{1,i\}}\right)$, $\sum_{j \in S^*_{B,i}} y^*_{B,i_j} = v(S^*_{B,i})$, and $(S^*_{B,i}, \bar{w}_1^{B,i})$ solves

$$\max_{(T,y_1) \in \rho_i(N) \times [\eta,1]} \left\{ \frac{[v(T) - y_1] F_B(\Omega_{B,y_1})}{1 + \{|T| - 2\}\delta} \right\}, \tag{1}$$

and i accepts a proposal at any continuation game $G^i(B)$ only if it offers her at least $\bar{a}_i^{B,i}$, where for all $k \in N \setminus \{1\}$,

$$\bar{a}_k^{B,k} = \frac{\delta \left[v(S^*_{B,k}) - \bar{w}_1^{B,k} \right] F_B \left(\Omega_{B,\bar{w}_1^{B,k}} \right)}{1 + \{|S^*_{B,k}| - 2\}\delta}.$$

Note that it is easy to see that the restriction (**a**) $|S| = |T| \implies v(S \cup \{1\}) = v(T \cup \{1\}), \forall\, S, T \subseteq N \setminus \{1\}$ implies that for all $j, k \neq 1$, $|S^*_{B,j}| = |S^*_{B,k}|$ and $y^*_{B,j} = y^*_{B,k}$. Hence, given this restriction, (**b**) $\forall\, j, k \neq 1$, $\bar{a}_j^{B,j} = \bar{a}_k^{B,k}$.

Now, let us consider the two possible kinds of proposals that i can make at this information set: *acceptable* (where the uninformed agents addressed by the proposal are supposed to accept it), and *unacceptable* (where at least one of uninformed parties addressed by the proposal rejects it). An acceptable proposal by i would propose a coalition T and a distribution y of its coalitional worth among its members such that every $j \in T \setminus \{i, 1\}$, $F_B(\Omega_{B,y_1}) y_j \geq \bar{a}_j^{B,j}$. This is because, by construction of $\succ_r^1$, the informed party responds last to a proposal. And so, while responding to any proposal involving some coalition S, any uninformed party $j \in S \setminus \{1, i\}$ who is prescribed to accept this proposal, *is still uncertain* about whether the informed party would accept the proposal in future or not. Hence, j would accept a proposal only if she is offered a sure amount y_j that leads to an expected payoff as great as $\bar{a}_j^{B,j}$.[19]

[19]Thus, offering greater amounts to the informed party actually decreases the amount that is needed

Note that the best acceptable proposal that i as a proposer can make at continuation game $G^i(B)$ is one that solves the following maximization problem:

$$\max_{(T,y_1)\in\rho_i(N)\times[\eta,1],1\in T}\left[\{v(T)-y_1\}F_B(\Omega_{B,y_1})-\sum_{j\neq T\setminus\{1,i\}}\bar{a}_j^{B,j}\right], \tag{2}$$

where objective function of (2) is the expected payoff to $i \neq 1$ by proposing to form a coalition T that contains 1, offering y_1 to 1 and $\bar{a}_j^{B,j}$ to all other agents j in T. By restriction **(a)**, and hence **(b)**, it follows that the worths of the coalitions chosen as solution to (2) by any $i, j \neq 1$ must be *identical*. Similarly, the amount y_1 to be offered to 1 chosen as a solution to (2) by any $i, j \neq 1$ must be same. Now, it is easy to see that the solution to (1) also solves (2) and vice-versa.[20] This implies that there is no profitable one-deviation of making an acceptable proposal available to any uninformed party $i \neq 1$ at any continuation game $G^i(B)$.

Further, making an unacceptable proposal is never optimal for i as it only causes a delay of a period without updating the beliefs held by uninformed parties, and passes the proposer power to some other agent. So the maximum possible expected payoff from making such an unacceptable proposal is $\bar{a}_i^{B,i}$. Note that, by construction, this is less than the expected payoff of $\frac{\bar{a}_i^{B,i}}{\delta}$ that i would have got had she proposed according to σ^*. Thus, we can infer that there is no profitable one-deviation available to any uninformed party at any information set where she has the move to propose.

Finally, consider any information set where the informed party has the move to respond to a proposal made by some uninformed agent $i \neq 1$. Irrespective of the amount offered by i's proposal to any type of the informed party 1_π, if 1_π one-deviates to reject and make a counteroffer, then, as prescribed in σ^*, she would make a non-informative unacceptable proposal in the next period. This would lead to an off the candidate equilibrium path information set (with respect to σ^*) where beliefs get revised as per Bayes' rule, to a suitable conditional distribution over $[\eta, \max\{\eta, \delta^2\bar{y}_1\}]$ where $\bar{y}_1$ is the amount prescribed to be offered to the informed party 1 by σ^* at this information set. By sequential rationality (and our regularity condition) $\bar{y}_1 \in [\eta, \max\{\eta, \delta^2\bar{y}_1\}]$ implying that $\bar{y}_1 = \eta$. Hence, such a unilateral one-deviation is not profitable for 1_π.

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to be offered to obtain agreement of an uninformed party to a proposal.

[20] A formal argument is as follows. Suppose there exists a solution (T^I, w_1^I) to (1), which does not solve (2). That is, for any (T^{II}, w_1^{II}) that solves (2), either $|T^I| \neq |T^{II}|$ or $w_1^I \neq w_1^{II}$ or both. Therefore, by restrictions **(a)** and **(b)**, for all $j \neq 1$, $\bar{a}_j^{B,j} = \frac{\delta\left[v(T^I)-w_1^I\right]F_B\left(\Omega_{B,w_1^I}\right)}{1+\{|T^I|-2\}\delta} \geq \frac{\delta\left[v(T^{II})-w_1^{II}\right]F_B\left(\Omega_{B,w_1^{II}}\right)}{1+\{|T^{II}|-2\}\delta}$. Now, $\left[\left\{v(T^{II})-w^{II}\right\}F_B(\Omega_{B,w^{II}})-\sum_{j\neq T^{II}\setminus\{1,i\}}\bar{a}_j^{B,j}\right] > \left[\left\{v(T^I)-w^I\right\}F_B(\Omega_{B,w^I})-\sum_{j\neq T^I\setminus\{1,i\}}\bar{a}_j^{B,j}\right]$, which implies that $\frac{[v(T^{II})-w^{II}]F_B(\Omega_{B,w^{II}})}{1+\{|T^{II}|-2\}\delta} > \frac{[v(T^I)-w^I]F_B(\Omega_{B,w^I})}{1+\{|T^I|-2\}\delta}$, which in turn contradicts our supposition that (T^I, w^I) solves (1).

Hence, σ^* constitutes an equilibrium. $\square$

Proof of (2): Fix any equilibrium σ' and any uninformed agent $i \neq 1$. Note that, by Corollary 1, each type of informed agent accepts a proposal on the equilibrium path if and only if she is offered an amount no less than her outside option. Further, Corollary 1 implies that: if i is the first proposer, then either the informed party 1 quits the game or else she accepts i's proposal. In either case the game ends in the first period, and there is no continuation game. For each uninformed party $j \neq 1$, let $\bar{x}_j^{[\eta,1],j}$ denote the equilibrium expected payoff of j, when she is the first proposer in our bargaining game and all agents play according to σ'. Define for all $j \neq 1$, $\tilde{a}_j^{[\eta,1],j} := \delta \bar{x}_j^{[\eta,1],j}$.

Now, as discussed in the proof of (1) above, making an acceptable proposal (given σ') is always more profitable to i than making an unacceptable proposal. Hence, σ' is an equilibrium only if: whenever i is the first proposer of our bargaining game, she proposes $(S'_{[\eta,1],i}, y'_{[\eta,1],i})$ where $\sum_{t \in S'_{[\eta,1],i}} y'_{[\eta,1],i_t} = v(S'_{[\eta,1],i})$, $y'_{[\eta,1],i_1} = w_1'^{[\eta,1],i}$, and $y'_{[\eta,1],i_j} = \frac{\tilde{a}_j^{[\eta,1],i}}{F_{[\eta,1]}\left(\Omega_{[\eta,1],y'_{[\eta,1],i_1}}\right)}$, $\forall\, j \in S'_{[\eta,1],i} \setminus \{1,i\}$ such that:[21]

$$(S'_{[\eta,1],i}, w_1'^{[\eta,1],i}) \text{ solves } \max_{(T,y_1) \in \rho_i(N) \times [\eta,1], 1 \in T} \left[\{v(T) - y_1\} F_{[\eta,1]}\left(\Omega_{[\eta,1],y_1}\right) - \sum_{j \neq T \setminus \{1,i\}} \tilde{a}_j^{[\eta,1],j} \right]. \tag{3}$$

Note that maximization problem (3) identifies the best possible acceptable proposal that any uninformed party $i \neq 1$ can make in the continuation game $G^i([\eta,1])$. Since, our bargaining protocol requires rejectors to propose next period, every active agent while responding, can always reject and propose the best possible acceptable proposal at the very next period. That is, for any agent j,

$$\tilde{a}_j^{[\eta,1],j} \geq \max_{(T,y_1) \in \rho_j(N) \times [\eta,1], 1 \in T} \delta \left[\{v(T) - y_1\} F_{[\eta,1]}(\Omega_{[\eta,1],y_1}) - \sum_{l \neq T \setminus \{1,i\}} \tilde{a}_l^{[\eta,1],l} \right].$$

Now, if σ' requires this weak inequality to be *strict*, then it prescribes rejection of proposals that offer j *more* expected payoff than what she can get by making the best possible acceptable proposal in the next period, which would not constitute equilibrium.

[21] Note that $y'_{[\eta,1],i}$ is a specific distribution of the worth of the coalition $S'_{[\eta,1],i}$, constructed in a manner that it is acceptable to all uninformed members of $S'_{[\eta,1],i}$ other than i. This construction depends on the relevant beliefs, and is described below for the information sets with prior beliefs.

Therefore, if σ' is an equilibrium then:

$$\tilde{a}_j^{[\eta,1],j} = \max_{(T,y_1)\in\rho_j(N)\times[\eta,1],1\in T} \delta\left[\{v(T)-y_1\}F_{[\eta,1]}(\Omega_{[\eta,1],y_1}) - \sum_{l\neq T\setminus\{1,i\}} \tilde{a}_l^{[\eta,1],l}\right] \text{ for all } j. \quad (4)$$

Now, as argued in proof of (1) above, the restriction (**a**) $|S| = |T| \implies v(S\cup\{1\}) = v(T\cup\{1\}), \forall S,T \subseteq N\setminus\{1\}$ implies that any solution to (1) also solves the maximization (3) and vice-versa. Hence, if σ' is an equilibrium then (as described in Remark 1) i must propose a $(S^*_{[\eta,1],i}, y^*_{[\eta,1],i})$ at the start of the game. By Theorem 3 and Corollary 1, any type of the informed party 1_π with $\pi \leq y^*_{[\eta,1],i_1} = \bar{w}_1^{[\eta,1],i}$ would accept it, while any other type would quit the game (as shown in Theorem 3). Finally, if any type of the informed party gets to propose in the first period of the game, then, as argued in Theorem 1, she makes an unacceptable proposal, which would get rejected by the the top ranked uninformed player (out of those to whom the unacceptable proposal was made) according $\succ_r^1$; who, in turn, proposes the aforementioned proposal $(S^*_{[\eta,1],i}, y^*_{[\eta,1],i})$ in the next period.

Since the equilibrium σ' was chosen arbitrarily, and since beliefs on the equilibrium path must be generated using Bayes' rule (by definition), the result follows. □

Remark 2. *Note that Theorem 4 characterizes a class of equilibria Σ^*, which is essentially unique in the sense that: any equilibria outside Σ^* has an analogue in Σ^* that generates the same equilibrium path for a given first proposer. This result becomes stronger if the underlying characteristic function, and the prior beliefs generate a unique solution to the problem* $\max\left\{\frac{[v(T)-y_1]F_B(\Omega_{B,y_1})}{1+\{|T|-2\}\delta} : (T,y_1) \in \rho_i(N)\times[\eta,1]\right\}$. *In that case, Σ^* becomes a singleton set, and so, Theorem 4 implies that any equilibrium must generate the same equilibrium path irrespective of the first proposer.*

The next result characterizes the equilibrium of our bargaining game, for a more general form of underlying cooperative game that need not be symmetric around the informed party 1. However, this absence of symmetry introduces new complications, which require us to first define the following algorithm, and then use it to characterize the equilibrium for general games (where for any $S,T \subseteq N\setminus\{1\}$, $|S| = |T|$ need not imply that $v(S\cup\{1\}) = v(T\cup\{1\})$).

Algorithm Θ^B: *Consider any information set where uninformed parties hold some (arbitrary) belief B - that is, believe the informed party's outside option to be distributed with a conditional distribution $F_B(.)$ over a support $\tilde{B} \in \mathcal{B}([0,1])$. Define a sequence of $\{(A_k^{*B}, w_k^{*B})\}_k$ of pairs of 'coalitions and reals' such that*

- (A^{*B}_{1}, w^{*B}_{1}) *solves*

$$\max_{(T,y_1)\in\ \{S:S\subseteq N\}\times[\eta,1]} \left\{ \frac{[v(T)-y_1]F_B(\Omega_{B,y_1})}{1+\{|T|-2\}\delta} \right\} \tag{5}$$

Define p^{*B}_1 *to be the maximized value of (5).*

- *For any* $k>1$*, define* $\mathcal{A}^B_{k-1} := \cup_{l=1}^{k-1} A^{*B}_l$*. If* $\mathcal{A}^B_{k-1} = N$ *then stop, or else construct* (A^{*B}_k, w^{*B}_k) *to be a solution to*

$$\max_{(T,y_1)\in\ \{S:S\subseteq N\}\times[\eta,1]} \frac{\left[\{v(T)-y_1\}F_B(\Omega_{B,y_1}) - \sum_{i\in T\cap\mathcal{A}_{k-1}} p^{*B}_i\right]}{1+\{|T\setminus\mathcal{A}_{k-1}|-2\}\delta} \tag{6}$$

Define p^{*B}_k *be the maximized value of (6). Also, define* $W^{*B} := \max\{w^{*B}_1, w^{*B}_2, \ldots\}$.[22]

Now, we use the algorithm $\Theta := \{\Theta^B\}_{\tilde{B}\in\mathcal{B}([0,1])}$ to define the following class of strategy-belief pairs $\Gamma^*(\succ^1_r)$.

Definition 2. $\Gamma^*(\succ^1_r)$ *is the set of strategy-belief pairs* θ^* *which prescribe that:*

1. *for any uninformed party* $i \neq 1$*, at any continuation game* $G^i(B)$*,*

   - *she accepts a proposal* (T,y) *if and only if all uninformed parties* $j\in T\setminus\{1\}$ *are offered a (sure) share of at least* $\frac{p^{*B}_{\underline{k}_j}}{F_B(\Omega_{B,y_1})}$ *(as defined in algorithm* Θ^B*) where for all* $t\in N\setminus\{1\}$*,* $\underline{k}_t := \min\{k\in\mathbb{N} : t\in A^{*B}_k\}$*.*
   - *she proposes* $(S^*_{B,i}, y^*_{B,i})$ *such that* $S^*_{B,i} = A^{*B}_{\underline{k}_i}$*,* $y^*_{B,i1} = w^{*B}_{\underline{k}_i}$ *and for all* $j\in S^*_{B,i}\setminus\{i,1\}$*,* $y^*_{B,ij} = \frac{p^{*B}_{\underline{k}_j}}{F_B\left(\Omega_{B,w^{*B}_{\underline{k}_i}}\right)}$.

2. *At any continuation game* $G^1(B)$*, all types of the informed party* 1_π *make an unacceptable proposal* (T^*_B, z^*_B) *such that: (i) the maximal agent in* $T^*_B\setminus\{1\}$ *according to* $\succ^1_r$*, say* $\overline{m}$*, satisfies the equation* $w^{*B}_{\underline{k}_{\overline{m}}} = W^{*B}$*, and (ii) there exists* $l\in T^*_B\setminus\{1\}$ *such that* $z^*_{Bl} < \frac{p^{*B}_{\underline{k}_l}}{F_B\left(\Omega_{B,z^*_{B1}}\right)}$*. Further, at any information set where, any type of the informed party* 1_π *has the move to respond, she accepts if it offers her an amount at least as great as* π *or else she quits.*

3. *At any off the prescribed path (by* θ^**) information set where Bayes' rule cannot be applied, if an uninformed* $i\neq 1$ *receives an acceptable proposal (as per the condition*

[22] Note that for any belief B, Θ^B may not generate a unique sequence $\left\{\left(A^{*B}_k, w^{*B}_k\right)\right\}$ as optimization problems (5) and (6) may have multiple solutions. The W^{*B}, however, is unique for a given belief B.

1 above) from the informed party 1_π, then she changes her beliefs to put full probability on the event that $\pi = \eta$. At any other off the prescribed path information set where Bayes' rule is not applicable, any uninformed party keeps her beliefs unchanged as those held at the previous period. At every other information set, beliefs are updated in accordance to Bayes' rule.

We show below that for the general class of cooperative games where uninformed parties may not be symmetric in nature, the aforementioned strategy-belief pair θ^* constructed using the algorithm Θ, would constitutes a generically unique equilibrium given out-of-equilibrium beliefs.

Theorem 5. *For any $\succ_r^1, \succ_p$:*

1. *any $\theta^* \in \Gamma^*(\succ_r^1)$ constitutes an equilibrium.*
2. *if there exists an equilibrium $\theta_e \notin \Gamma^*(\succ_r^1)$, then there exists a corresponding $\theta_e^* \in \Gamma^*(\succ_r^1)$ such that θ_e and θ_e^* prescribe the same actions and beliefs, at all information sets on the equilibrium path.*

Proof: See Appendix. □

Remark 3. *As argued in Remark 2, for any given $\succ_r^1$, Theorem 5 characterizes an essentially unique class of equilibria $\Gamma^*(\succ_r^1)$, where any equilibrium outside $\Gamma^*(\succ_r^1)$ has an analogue in $\Gamma^*(\succ_r^1)$, which generates the same equilibrium path for a given first proposer. Also, as before, the result becomes stronger if the underlying characteristic function, and the prior beliefs generate unique solutions to (5) and (6) in algorithm Θ^{B_0}. In that case, $\Gamma^*(\succ_r^1)$ becomes a singleton set, and so, Theorem 5 implies that any equilibrium must generate the same equilibrium path.*

In the following corollary, we present sufficient condition for formation of the grand coalition on the equilibrium path.

Corollary 2. *Whenever the prior belief is a uniform distribution over $[\eta, 1]$: if $\frac{v(N)}{|N|-1} > \frac{v(S)}{|S|-1}, \forall\, S \subsetneq N$ then the unique equilibrium outcome of our bargaining game is formation of the grand coalition N. Further,*

- *if 1 is the first proposer, on the equilibrium path, 1 proposes $\{1, \tilde{j}\}$ to any uninformed agent $\tilde{j}$, offering her an amount strictly less than $\frac{2\delta(1-\eta)}{1+(|N|-2)\delta}$.*[23] *Agent $\tilde{j}$ rejects this proposal, and proposes in the next period the grand coalition N, offering all other uninformed agents the amount $\frac{2\delta(1-\eta)}{1+(|N|-2)\delta}$, offering 1 the amount $\frac{1+\eta}{2}$, and this proposal is accepted with probability $\frac{1}{2}$ (or else the game ends). The expected exante equilibrium payoff of $\tilde{j}$ is $\frac{(1-\eta)}{1+(|N|-2)\delta}$, while that of other uninformed agents is $\frac{\delta(1-\eta)}{1+(|N|-2)\delta}$.*

[23] She could propose any other proposal that would be unacceptable to $\tilde{j}$.

- *if some uninformed party $j \neq 1$ is the first proposer, on the equilibrium path, she proposes the grand coalition N offering all other uninformed agents the amount $\frac{2\delta(1-\eta)}{1+(|N|-2)\delta}$, offering 1 the amount $\frac{1+\eta}{2}$, and this proposal is accepted with probability $\frac{1}{2}$ (or else the game ends). The expected exante equilibrium payoff of j is $\frac{(1-\eta)}{1+(|N|-2)\delta}$, while that of other uninformed agents is $\frac{\delta(1-\eta)}{1+(|N|-2)\delta}$.*

Proof: It is easy to see that this result follows from statement (2) of Theorem 5 if $\mathcal{A}_1^{[\eta,1]} = N$. Note that given the prior belief of uniform distribution over $[\eta, 1]$, for any $x \in [\eta, 1]$, the probability of the informed party 1 having an outside option less than or equal to x, is $\frac{x-\eta}{1-\eta}$.

Now, fix any such $x \in (\eta, 1]$, and note that for any $S \subsetneq N$ with $v(S) > x$:

$$\frac{(v(N)-x)(x-\eta)}{(v(S)-x)(x-\eta)} = \frac{(v(N)-x)}{(v(S)-x)} > \frac{v(N)}{v(S)} > \frac{|N|-1}{|S|-1},^{24} \tag{7}$$

and so, for any $\delta \in (0, 1)$, we have

$$\frac{(|N|-1)}{(|S|-1)} > \frac{1+(|N|-2)\delta}{1+(|S|-2)\delta}.^{25} \tag{8}$$

Now, since $x \in (\eta, 1]$ and $\delta \in (0, 1)$ were arbitrarily chosen, from (7) and (8), it follows that:

$$\textbf{(E)} \quad \frac{[v(N)-x]\left(\frac{x-\eta}{1-\eta}\right)}{1+(|N|-2)\delta} > \frac{[v(S)-x]\left(\frac{x-\eta}{1-\eta}\right)}{1+(|S|-2)\delta}, \ \forall\, S \subset N \text{ with } |S| \geq 2, \ \forall\, x \in (\eta, 1]$$

By the maximization problem (5), **(E)** implies that $\mathcal{A}_1^{[\eta,1]} = N$. And so, it follows from statement (2) of Theorem 5 that $w{*}_1^{[\eta,1]}$ must solve (5) for the prior beliefs. Therefore, $w{*}_1^{[\eta,1]}$ must solve the maximization problem $\max\limits_{x\in[\eta,1]} g(x)$ where:

$$g(x) := \frac{\delta[1-x]\left(\frac{x-\eta}{1-\eta}\right)}{1+(|N|-2)\delta}, \forall\, x \in [\eta, 1].$$

It is easy to see that $w{*}_1^{[\eta,1]} = \frac{1+\eta}{2}$, $g''(\frac{1+\eta}{2}) < 0$, and $p{*}_1^{[\eta,1]} = \frac{\delta(1-\eta)/4}{1+(|N|-2)\delta}$. Further, given our prior belief, it is easy to see that the offer $\frac{1+\eta}{2}$ would be accepted by the informed party with probability $\frac{1}{2}$. Hence, the result follows. □

[24] The first inequality follows from the fact that any fraction greater than 1, sees a fall in its value if the numerator and the denominator are increased by the same amount. The second inequality follows from our supposition.

[25] It can easily be seen that any function $h(\delta) := \frac{1+b\delta}{1+a\delta}$ is increasing over $[0, 1]$ when $b > a$ (as $h'(\delta)$ is positive over $[0, 1]$).

4 Discussion

4.1 The regularity condition

There is a possibility that at some information sets of our bargaining game; the private information is believed to be too small, that is, range of the support of beliefs held by uninformed parties is too small to affect their proposal decisions. This is a peculiarity that arises out of the alternating offers nature of our game where the uninformed parties must offer an agent 1_0 an amount greater than 0, even when they are *certain* that $\pi = 0$. This is because, any type of agent 1 has the option to reject a proposal and make a counteroffer proposing the same proposal with a slightly reduced share offered to the previous proposer, and a slightly greater share demanded for herself. Since all agents derive disutility from delay, any uninformed party, say j, would choose to avoid such delay on the equilibrium path by offering agent 1_π the amount

$$\max\{\pi, \beta_j\} \text{ where } \beta_j := \max_{S \subseteq N, j \in S} \frac{\delta v(S)}{1 + (|S| - 1)\delta},$$

even when j knows 1_π's outside option to be π for sure. Therefore, at any information set where uninformed parties believe $\pi \in [a, b]$ with $b < \min_{j \neq 1} \beta_j$, any such type of informed agent, *irrespective of her outside option*, would find it profitable to make an acceptable proposal $(S^*, \bar{y})$ such that S^* solves $\max_{S \subseteq N} \frac{\delta v(S)}{1+(|S|-1)\delta}$ and $\bar{y}_k$ is equal to the maximized value for all $k \in S^* \setminus \{1\}$.[26] Our regularity condition $\mathcal{R}$ rules out such information sets where incompleteness of information about outside option of the informed party, which is the core issue of our paper, fails to affect equilibrium decisions in any manner.

4.2 A model variation

Note that our bargaining game allows agents to quit the game only while responding to an offer. We discuss below a variant of our game where the agents can only quit while proposing.

Suppose that the informed party is the first proposer in this altered game. Let x^* be the highest acceptable demand that 1 can make in any equilibrium. Then for all $\pi \leq x^*$, agent 1_π will demand x^* in the first period, while all types with $\pi > x^*$ would quit in the first period. In case, the informed party deviates by demanding an amount in excess of x^*, assume that beliefs remain unchanged at this information set (off the equilibrium path), leading to this offer being rejected by at least one of the uninformed parties in the proposed coalition, after which the rejector would make the equilibrium proposal prescribed to her. On the other hand, if the informed party is responding to a proposal that offers her an amount y_1, she will accept if $y_1 \geq \delta \max\{x^*, \pi\}$. In case, she rejects, the

[26] The proof follows from Chatterjee, Dutta, Ray and Sengupta [5].

informed party will quit if $\pi > x$, or else demand the amount x^*. Thus, the equilibrium path will not last any longer than two periods.

4.3 Out-of-equilibrium beliefs

We use out-of -equilibrium beliefs in Theorems 4 and 5, essentially to rule out the informed party making a potentially informative rejected offer. We have, however, shown that an equilibrium in which such an (informative) offer is made is impossible in any stationary PBE of the game. Thus, in our model, these beliefs are used to construct the equilibrium but we cannot specify beliefs that will give rise to the opposite conclusion in any equilibrium.

5 Appendix

5.1 Proof of Theorem 5

Proof of (1): Fix any uninformed party $i \neq 1$. By construction of θ^*, at any continuation game $G^i(B)$, i is prescribed to propose some $A^{*B}_{\underline{k}_i}$ (as defined in Θ^B), offer the informed party 1 the amount $w^{*B}_{\underline{k}_i}$ (again, as defined in Θ^B), and offer to all other uninformed agents $j \in A^{*B}_{\underline{k}_i}$ the amount $\frac{p^{*B}_{\underline{k}_j}}{F_B\left(\Omega_{B,w^{*B}_{\underline{k}_i}}\right)}$.

Note that, as argued in proof of (1) of Theorem 4, making an acceptable proposal (where all uninformed agents addressed by it, accept) is more profitable to i than making an unacceptable proposal (where θ^* prescribes at least one of the uninformed parties addressed by it to reject) at all relevant information sets, as the latter would cause a period of delay without updating any beliefs. Therefore, the best proposal that i can make when all other agents are playing according to θ^* must solve the problem:

$$\max_{\substack{(T,y_1)\in\rho_i(N)\times[\eta,1],\\ 1\in T}} \left\{ v(T) - y_1 - \sum_{\substack{j\in T\setminus\{1,i\},\\ j\notin\mathcal{A}^B_{\underline{k}_i-1}}} \frac{p^{*B}_{\underline{k}_j}}{F_B(\Omega_{B,y_1})} - \sum_{\substack{j\in T\setminus\{1,i\},\\ j\in\mathcal{A}^B_{\underline{k}_i-1}}} \frac{p^{*B}_{\underline{k}_j}}{F_B(\Omega_{B,y_1})} \right\} F_B(\Omega_{B,y_1}), \tag{9}$$

where objective function of (9) is the expected payoff to i by (acceptably) proposing to form a coalition T that contains 1, while offering y_1 to 1 and $\frac{p^*B_{\underline{k}_j}}{F_B(\Omega_{B,y_1})}$ to all other uninformed agents j in T. Note that by construction of algorithm Θ^B,

$$p^{*B}_{\underline{k}_l} = \frac{\delta\left[\left\{v(A^{*B}_{\underline{k}_l}) - w^{*B}_{\underline{k}_l}\right\} F_B\left(\Omega_{B,w^{*B}_{\underline{k}_l}}\right) - \sum_{t\in A^{*B}_{\underline{k}_l}\cap\mathcal{A}^B_{\underline{k}_l-1}} p^{*B}_{\underline{k}_t}\right]}{1+\left(|A^{*B}_{\underline{k}_l}\setminus\mathcal{A}^B_{\underline{k}_l-1}|-2\right)\delta}, \forall\, l \neq 1 \tag{10}$$

where $\mathcal{A}_0^B := \emptyset$. Further, by construction of Θ^B; for each $j \in A^{*B}_{\underline{k}_i} \setminus [\mathcal{A}^B_{\underline{k}_i - 1} \cup \{1, i\}]$, $p^{*B}_{\underline{k}_i} = p^{*B}_{\underline{k}_j} := P'$ (say), and $A^{*B}_{\underline{k}_i} = A^{*B}_{\underline{k}_j} := A'$ (say). Therefore, by construction, θ^* prescribes i to propose A' at the start of continuation game $G^i(B)$, offering the informed party $w^{*B}_{\underline{k}_i}$, and other uninformed members of A' the amount P'.

Now, suppose that $(A', w^{*B}_{\underline{k}_i})$ does not solve the maximization problem (9). Therefore, there exists a tuple (S', y_1') that solves (9); but either $S' \neq A'$ or $y_1' \neq w^{*B}_{\underline{k}_i}$ or both.[27] Note that by (10) and construction of Θ^B,

$$P' \geq \frac{\delta \left[\{v(S') - y_1'\} F_B\left(\Omega_{B, y_1'}\right) - \sum\limits_{t \in S' \cap \mathcal{A}^B_{\underline{k}_i - 1}} p^{*B}_{\underline{k}_t} \right]}{1 + (|S' \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2)\delta},$$

and so, from (9) it follows that:

$$\begin{aligned}
& \{v(S') - y_1'\} F_B(\Omega_{B, y_1'}) - \frac{\left(|S' \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2\right)\delta \left[\left\{v(S') - y_1'\right\} F_B\left(\Omega_{B, y_1'}\right) - \sum\limits_{t \in S' \cap \mathcal{A}^B_{\underline{k}_i - 1}} p^{*B}_{\underline{k}_t} \right]}{1 + (|S' \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2)\delta} - \sum_{\substack{t \in S' \setminus \{1, i\}, \\ t \in \mathcal{A}^B_{\underline{k}_i - 1}}} p^{*B}_{\underline{k}_t} \\
\geq \quad & \{v(S') - y_1'\} F_B(\Omega_{B, y_1'}) - \left[|S' \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2 \right] P' - \sum_{\substack{t \in S' \setminus \{1, i\}, \\ t \in \mathcal{A}^B_{\underline{k}_i - 1}}} p^{*B}_{\underline{k}_t} \\
> \quad & \left\{ v(A^{*B}_{\underline{k}_i}) - w^{*B}_{\underline{k}_i} \right\} F_B(\Omega_{B, w^{*B}_{\underline{k}_i}}) - \left[|A^{*B}_{\underline{k}_i} \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2 \right] P' - \sum_{\substack{t \in A^{*B}_{\underline{k}_i} \setminus \{1, i\}, \\ t \in \mathcal{A}^B_{\underline{k}_i - 1}}} p^{*B}_{\underline{k}_t}
\end{aligned}$$

which implies that:[28]

$$\frac{[v(S') - y_1'] F_B(\Omega_{B, y_1'}) - \sum\limits_{\substack{t \in S' \setminus \{1, i\}, \\ t \in \mathcal{A}^B_{\underline{k}_i - 1}}} p^{*B}_{\underline{k}_t}}{1 + \{|S' \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2\}\delta} > \frac{\left[v(A^{*B}_{\underline{k}_i}) - w^{*B}_{\underline{k}_i} \right] F_B\left(\Omega_{B, w^{*B}_{\underline{k}_i}}\right) - \sum\limits_{\substack{t \in A^{*B}_{\underline{k}_i} \setminus \{1, i\}, \\ t \in \mathcal{A}^B_{\underline{k}_i - 1}}} p^{*B}_{\underline{k}_t}}{1 + \{|A^{*B}_{\underline{k}_i} \setminus \mathcal{A}^B_{\underline{k}_i - 1}| - 2\}\delta},$$

which violates (6) in the construction of Θ^B, and thus, leads to a contradiction. Therefore, whenever all players other than i play according to θ^*, there is no profitable unilateral one-deviation from θ^* available to i, at any information set where she has the move to propose.

Now, note that the response prescriptions for the informed party 1 as well as the off the prescribed path beliefs of θ^*, are the same as that of σ^* in Definition 1. Further, as in σ^*,

[27] Note that for any possible belief B, $F_B(.)$ is non-decreasing and bounded in y_1, and so, the fact $|N| < \infty$ implies that a solution to (9) always exists.

[28] Note that our supposition implies strict inequality in the last inequality mentioned above.

all types of the informed party make a non-informative unacceptable proposal. Therefore, as argued in proof of 1 of Theorem 4, for any type of the informed party 1_π, making a unilateral one-deviation of rejecting a proposal (and hence, making a counteroffer), leads to Bayes' revision of uninformed parties' belief that $\pi = \eta$ - implying that 1_π would be offered η two periods after the one-deviation. Thus, such a one-deviation is not profitable.

Finally, at any information set where 1_π has the move to propose: if she unilaterally one-deviates to make an acceptable proposal, then arguing as in proof of (1) of Theorem 4, we can show that such a deviation is not profitable. On the other hand, if 1_π unilaterally one-deviates by making an unacceptable proposal different from the one prescribed in θ^*, then such a proposal either: (i) involves a two member coalition $\{1, j\}$ where ${w^*}^B_{\underline{k}_j} = {W^*}^B$, or (ii) it involves a two member coalition $\{1, j\}$ where ${w^*}^B_{\underline{k}_j} \neq {W^*}^B$, or else (iii) it involves a coalition comprising of more than two members. All these cases (i), (ii) and (iii) lead to off the equilibrium path information sets where Bayes' rule cannot be applied. As prescribed in θ^*, the beliefs remain unaffected by these one-deviations, and so, when all other agents are playing as per θ^*, the expected equilibrium payoffs to 1_π post such deviations, are not greater than that obtained by not deviating from θ^*. Thus, the informed party 1 has no profitable unilateral one-deviation at any information set where she has the move to propose. Hence, the result follows. □

Proof of (2): Fix any equilibrium θ' and any uninformed agent $i \neq 1$. Note that, by Corollary 1, each type of informed agent accepts a proposal on the equilibrium path if and only if she is offered an amount in excess of her outside option. Further, Corollary 1 implies that: if i is the first proposer, then either the informed party 1 accepts i's proposal or else quits the game in the very first period. Thus, the bargaining game ends in the first period, and hence, there is no continuation game on the equilibrium path other than $G^i(B_0)$, where B_0 is the common prior belief held over $[\eta, 1]$. For each uninformed party $j \neq 1$, let ${x'}_j^{B_0,j}$ denote the equilibrium expected payoff of j, when she is the first proposer in our bargaining game, and all agents play according θ'. That is, ${x'}_j^{B_0,j}$ is j's equilibrium payoff in the continuation game $G^j(B_0)$ when all agents play θ'. Define for all $j \neq 1$, ${a'}_j^{B_0,j} := \delta {x'}_j^{B_0,j}$

Now, as discussed in proof of statement (2) of Theorem 4 above, making an acceptable proposal (with respect to θ') is always more profitable to i than making an unacceptable proposal. Note that the best acceptable proposal which i can make must solve the problem:

$$\max_{\substack{(T,y_1)\in\rho_i(N)\times[\eta,1],\\ T\ni 1}} \left[\{v(T) - y_1\} F_{B_0}(\Omega_{B_0,y_1}) - \sum_{j\in T\setminus\{1,i\}} {a'}_j^{B_0,j} \right], \tag{11}$$

where objective function of (11) is the expected payoff to i by proposing to form a coalition

T that contains 1 (and i) while offering y_1 to 1 and $\frac{a'^{B_0,j}_j}{F_{B_0}(\Omega_{B_0,y_1})}$ to all other uninformed agents j in T.

Suppose that for each uninformed agent $j \neq 1$, θ' prescribes j to propose an acceptable proposal $(\bar{T}_j, \bar{y}_j)$ in the continuation game $G^j(B_0)$. Therefore, as argued in the proof of statement (2) in Theorem 4, we get that for all $j \neq 1$,

$$a'^{B_0,j}_j = \delta\left\{v(\bar{T}_j) - \bar{y}_{j1}\right\} F_{B_0}(\Omega_{B_0,\bar{y}_{j1}}) - \delta \sum_{k \neq \bar{T}_j \setminus \{1,j\}} a'^{B_0,k}_k. \tag{12}$$

Hence, for all $j \neq 1, i$,

$$a'^{B_0,j}_j = \delta\left\{v(\bar{T}_j) - \bar{y}_{j1}\right\} F_{B_0}(\Omega_{B_0,\bar{y}_{j1}}) - \delta \sum_{k \neq \bar{T}_j \setminus \{1,i\}} a'^{B_0,k}_k + \delta a'^{B_0,j}_j - \delta a'^{B_0,i}_i$$

implying that:

$$a'^{B_0,j}_j = \delta\left[\left\{v(\bar{T}_j) - \bar{y}_{j1}\right\} F_{B_0}(\Omega_{B_0,\bar{y}_{j1}}) - \sum_{k \neq \bar{T}_j \setminus \{1,i\}} a'^{B_0,k}_k\right] + \left(a'^{B_0,j}_j - a'^{B_0,i}_i\right)\delta. \tag{13}$$

Now, by (12), for all $i \neq 1, j$, $a'^{B_0,i}_i \geq \delta\left\{v(\bar{T}_j) - \bar{y}_{j1}\right\} F_{B_0}(\pi \leq \bar{y}_{j1}) - \delta \sum_{k \neq \bar{T}_j \setminus \{1,i\}} a'^{B_0,k}_k$, and so, from (13), we get that: $\left(a'^{B_0,j}_j - a'^{B_0,i}_i\right) \leq \delta\left(a'^{B_0,j}_j - a'^{B_0,i}_i\right)$ for all $i \in \bar{T}_j \setminus \{1, j\}$. Since $\delta \in (0,1)$, this inequality implies that

$$(\mathbf{A})\ \ a'^{B_0,j}_j \leq a'^{B_0,i}_i \text{ for all } j \neq 1 \text{ and all } i \in \bar{T}_j \setminus \{1, j\}.$$

Now, from **(A)** it follows that for all $j \neq 1$, $a'^{B_0,j}_j = \delta\left\{v(\bar{T}_j) - \bar{y}_{j1}\right\} F_{B_0}(\Omega_{B_0,\bar{y}_{j1}}) - \delta \sum_{i \in \bar{T}_j \setminus \{1,i\}} a'^{B_0,i}_i \leq \delta\left[\left\{v(\bar{T}_j) - \bar{y}_{j1}\right\} F_{B_0}(\Omega_{B_0,\bar{y}_{j1}}) - \left(|\bar{T}_j| - 2\right) a'^{B_0,j}_j\right]$, which, by construction of $\left(A^{*B_0}_1, w^{*B_0}_1\right)$ in Θ^B,[29] implies that:

$$(\mathbf{B})\ a'^{B_0,j}_j \leq \frac{\delta\left[v(\bar{T}_j) - \bar{y}_{1j}\right] F_{B_0}\left(\Omega_{B_0,\bar{y}_{1j}}\right)}{1 + \left(|\bar{T}_j| - 2\right)\delta} \leq \frac{\delta\left[v(A^{*B_0}_1) - w^{*B_0}_1\right] F_{B_0}\left(\Omega_{B_0,w^{*B_0}_1}\right)}{1 + \left(|A^{*B_0}_1| - 2\right)\delta}, \forall\, j \neq 1.$$

In the following part of the proof, we will use the inequalities **(A)** and **(B)** to establish that $(\bar{T}_j, \bar{y}_j)$ must involve one of the pairs $\left(A^{*\ B_0}_{\underline{k}_j}, w^{*\ B_0}_{\underline{k}_j}\right)$ constructed using Θ^{B_0}, in the following manner:

$$\bar{T}_j = A^{*\ B_0}_{\underline{k}_j},\ \sum_{t \in S'} y'_t = v(S'),\ \bar{y}_{j1} = w^{*\ B_0}_{\underline{k}_j},\ \text{and } \bar{y}_{jt} = \frac{p^{*B_0}_1}{F_{B_0}\left(\Omega_{B_0,w^{*B_0}_1}\right)}, \forall\, t \in \bar{T}_j \setminus \{1, j\}.$$

[29]See equation (5).

Given construction of $\Gamma^*(\succ_r^1)$, this will establish the result.

We begin by focussing on the collection of uninformed parties in $\mathcal{A}_1^{B_0}$ (as defined by Θ^{B_0}). Suppose that there exists an agent $j' \in \mathcal{A}_1^{B_0}$ such that θ' prescribes her an acceptable proposal $(\bar{T}_{j'}, \bar{y}_{j'})$ at the start of continuation game $G^{j'}(B_0)$ such that it is not consistent with the algorithm Θ^{B_0}. That is, $(\bar{T}_{j'}, \bar{y}_{j'_1})$ does not solve (5). In other words,

$$(\mathbf{C}) \quad (\bar{T}_{j'}, \bar{y}_{j'_1}) \notin argmax \left\{ \frac{\delta[v(T) - y_1]F_B(\Omega_{B,y_1})}{1 + \{|T| - 2\}\delta} : (T, y_1) \in \ \{S : S \subseteq N\} \times [\eta, 1] \right\}.$$

Note that,

$$a'^{B_0,j'}_{j'} \geq \delta \left[\left\{ v(A^{*B_0}_1) - w^{*B_0}_1 \right\} F_{B_0} \left(\Omega_{B_0, w^{*B_0}_1} \right) - \sum_{i \in A^{*B_0}_1 \setminus \{1,i\}} a'^{B_0,i}_i \right].$$

By **(B)**, the inequality above implies that

$$\begin{aligned}
&\frac{\delta \left[v(\bar{T}_{j'}) - \bar{y}_{j'_1} \right] F_{B_0} \left(\Omega_{B_0, y_{j'_1}} \right)}{1 + \left(|T^*_{j'}| - 2 \right) \delta} \geq a'^{B_0,j'}_{j'} \\
\geq \ & \delta \left[\left\{ v(A^{*B_0}_1) - w^{*B_0}_1 \right\} F_{B_0} \left(\Omega_{B_0, w^{*B_0}_1} \right) - \frac{\delta \left(|A^{*B_0}_1| - 2 \right) \left[v(A^{*B_0}_1) - w^{*B_0}_1 \right] F_{B_0} \left(\Omega_{B_0, w^{*B_0}_1} \right)}{1 + \left(|A^{*B_0}_1| - 2 \right) \delta} \right] \\
= \ & \frac{\delta \left[v(A^{*B_0}_1) - w^{*B_0}_1 \right] F_{B_0} \left(\Omega_{B_0, w^{*B_0}_1} \right)}{1 + \left(|A^{*B_0}_1| - 2 \right) \delta},
\end{aligned}$$

which contradicts **(C)**. Thus, we get that for all $j \in \mathcal{A}_1^{B_0}$, the best acceptable proposal that j is prescribed by θ' to make at start of $G^j(B_0)$ must involve some $(A^{*B_0}_1, w^{*B_0}_1)$ that solves (5).[30] Further, for all $j \in \mathcal{A}_1^{B_0}$,

$$a'^{B_0,j}_j = \frac{\delta \left[v(A^{*B_0}_1) - w^{*B_0}_1 \right] F_{B_0} \left(\Omega_{B_0, w^{*B_0}_1} \right)}{1 + \left(|A^{*B_0}_1| - 2 \right) \delta} = p^{*B_0}_1.$$

Now fix any natural number $L > 1$ such that $\mathcal{A}_L^{B_0} \neq N$, and suppose that all $j \in \mathcal{A}_L^{B_0}$ propose $A^{*B_0}_{\underline{k}_j}$ offering informed agent 1 the amount $w^{*B_0}_{\underline{k}_j}$, and offering all other uninformed agents in $A^{*B_0}_{\underline{k}_j}$ the amount $p^{*B_0}_{\underline{k}_i} / F_{B_0}(\Omega_{B_0, w^{*B_0}_{\underline{k}_j}})$, at the start of $G^j(B_0)$. Given this supposition, we present a proof by induction below, which shows that all uninformed

[30]That is, all agents in $\mathcal{A}_1^{B_0}$ propose the coalition $A^{*B_0}_1$, offer $w^{*B_0}_1$ to the informed party, and offer all other uninformed agents $i \in A^{*B_0}_1 \setminus \{1, i\}$ the amount $p^{*B_0}_1 / F_{B_0}(\Omega_{B_0, w^{*B_0}_1})$.

agents in $j \in \mathcal{A}^{B_0}_{L+1}$ must propose in the same manner at the start of $G^j(B_0)$. Note that, by construction, $\mathcal{A}^{B_0}_{L} \subseteq \mathcal{A}^{B_0}_{L+1}$, and hence, we only need to consider agents in $\mathcal{A}^{B_0}_{L+1} \setminus \mathcal{A}^{B_0}_{L}$.

So, fix any $j \in \mathcal{A}^{B_0}_{L+1} \setminus \mathcal{A}^{B_0}_{L}$. As argued earlier, at start of $G^j(B_0)$, the best acceptable proposal that j can make involves a pair $(\hat{T}_j, \hat{y}_{j_1})$ that solves (11). Hence, using our induction hypothesis, we get that:

$$a'^{B_0,j}_j = \delta \left[\left\{ v(\hat{T}_j) - \hat{y}_{j_1} \right\} F_{B_0}\left(\Omega_{B_0,\hat{y}_{j_1}}\right) - \sum_{t \in \hat{T}_j \setminus [\{1,j\} \cup \mathcal{A}^{B_0}_L]} a'^{B_0,t}_t - \sum_{t \in [\hat{T}_j \setminus \{1,j\}] \cap \mathcal{A}^{B_0}_L} p^{*B_0}_{\underline{k}_t} \right]$$

As before, using **(A)** we get that:

$$\begin{aligned}
\textbf{(D)} \quad a'^{B_0,j}_j \quad & \leq \quad \frac{\delta \left[\left\{ v(\hat{T}_j) - \hat{y}_{j_1} \right\} F_{B_0}\left(\Omega_{B_0,\hat{y}_{j_1}}\right) - \sum_{t \in [\hat{T}_j \setminus \{1,j\}] \cap \mathcal{A}^{B_0}_L} p^{*B_0}_{\underline{k}_t} \right]}{1 + \left(|\bar{T}_j \setminus \mathcal{A}^{B_0}_L| - 2\right)\delta} \\
& \leq \quad \frac{\delta \left[\left\{ v(A^{*B_0}_{L+1}) - w^{*B_0}_{L+1} \right\} F_{B_0}(\Omega_{B_0,w^{*B_0}_{L+1}}) - \sum_{t \in [\hat{T}_j \setminus \{1,j\}] \cap \mathcal{A}^{B_0}_L} p^{*B_0}_{\underline{k}_t} \right]}{1 + \left(|A^{*B_0}_{L+1} \setminus \mathcal{A}^{B_0}_L| - 2\right)\delta}.
\end{aligned}$$

Now, as before, suppose that $(\hat{T}_j, \hat{y}_{j_1})$ does not solve (6). Therefore, by **(D)**:

$$\frac{\delta\left[\left\{v(\hat{T}_j) - \hat{y}_{j_1}\right\} F_{B_0}\left(\Omega_{B_0,\hat{y}_{j_1}}\right) - \sum_{t\in[\hat{T}_j\setminus\{1,j\}]\cap\mathcal{A}_L^{B_0}} p^{*B_0}_{\underline{k}_t}\right]}{1+\left(|\hat{T}_j\setminus\mathcal{A}_L^{B_0}|-2\right)\delta}$$

$$\begin{aligned}
\geq\ & a'^{B_0,j}_j \\
\geq\ & \delta\left[v(A^{*B_0}_{L+1}) - w^{*B_0}_{L+1}\right] F_{B_0}\left(\Omega_{B_0,w^{*B_0}_{L+1}}\right) - \delta \sum_{t\in[\hat{T}_j\setminus\{1,j\}]\cap\mathcal{A}_L^{B_0}} p^{*B_0}_{\underline{k}_t} \\
& - \frac{\delta^2\left(|A^{*B_0}_{L+1}\setminus\mathcal{A}_L^{B_0}|-2\right)\left[\left\{v(A^{*B_0}_{L+1}) - w^{*B_0}_{L+1}\right\} F_{B_0}\left(\Omega_{B_0,w^{*B_0}_{L+1}}\right) - \sum_{t\in[\hat{T}_j\setminus\{1,j\}]\cap\mathcal{A}_L^{B_0}} p^{*B_0}_{\underline{k}_t}\right]}{1+\left(|A^{*B_0}_{L+1}\setminus\mathcal{A}_L^{B_0}|-2\right)\delta}
\end{aligned}$$

$$\Longleftrightarrow \quad \frac{\delta\left[\left\{v(\hat{T}_j) - \hat{y}_{j_1}\right\} F_{B_0}\left(\Omega_{B_0,\hat{y}_{j_1}}\right) - \sum_{t\in[\hat{T}_j\setminus\{1,j\}]\cap\mathcal{A}_L^{B_0}} p^{*B_0}_{\underline{k}_t}\right]}{1+\left(|\hat{T}_j\setminus\mathcal{A}_L^{B_0}|-2\right)\delta}$$

$$\geq \quad \frac{\delta\left[\left\{v(A^{*B_0}_{L+1}) - w^{*B_0}_{L+1}\right\} F_{B_0}\left(\Omega_{B_0,w^{*B_0}_{L+1}}\right) - \sum_{t\notin[\hat{T}_j\setminus\{1,j\}]\cap\mathcal{A}_L^{B_0}} p^{*B_0}_{\underline{k}_t}\right]}{1+\left(|A^{*B_0}_{L+1}\setminus\mathcal{A}_L^{B_0}|-2\right)\delta},$$

which implies that $(\hat{T}_j, \hat{y_1}_j)$ solves (6). Thus, we get a contradiction. Thus, by the induction proof we have established that for any uninformed party $i \neq 1$, the equilibrium path generated by any θ' in $G^i(B_0)$, is the same as that generated by *some* $\theta^* \in \Gamma^*(\succ^1_r)$. Since, beliefs are generated on the equilibrium path using Bayes' rule, the result follows whenever an uninformed party is the first proposer in our bargaining game.

Now, consider the only remaining possibility that the informed agent 1 is the first proposer. If that is the case, then, by Theorem 1, she makes an unacceptable proposal which is subsequently rejected. However, such an unacceptable proposal passes the proposer power to the highest ranked uninformed member of her proposed coalition as per $\succ^1_r$, and this uninformed agent gets the first chance to respond to 1's proposal. So the informed party, at the start of $G^1(B_0)$, can do no better than making an unacceptable proposal that passes the proposer power to an uninformed party $\tilde{j}$ such that $p^{*B_0}_{\underline{\tilde{j}}} = W^{*B_0}$. Therefore, by construction of $\Gamma^*(\succ^1_r)$, and our inference from the previous paragraph, the result follows when the informed party 1 is the first proposer in our bargaining game. $\square$

References

[1] D. Abreu and F. Gul. Bargaining and reputation. *Econometrica*, 68:85–117, 2000.

[2] A. Admati and M. Perry. Strategic delay in bargaining. *The Review of Economic Studies*, 54:345–364, 1987.

[3] L. Ausubel and R. Deneckere. Bargaining and the right to remain silent. *Econometrica*, 60:597–625, 1992.

[4] K. Binmore, A. Rubinstein, and A. Wolinsky. The Nash bargaining solution in economic modelling. *The RAND Journal of Economics*, 17:176–188, 1986.

[5] K. Chatterjee, B. Dutta, D. Ray, and K. Sengupta. A noncooperative theory of coalitional bargaining. *The Review of Economic Studies*, 60:463–477, 1993.

[6] K. Chatterjee and H. Sabourian. Multiperson bargaining and strategic complexity. *Econometrica*, 68:1491–1509, 2000.

[7] K. Chatterjee and L. Samuelson. Bargaining with two-sided incomplete information: an infinite horizon model with alternating offers. *The Review of Economic Studies*, 54:175–192, 1987.

[8] K. Chatterjee and L. Samuelson. Bargaining under two-sided incomplete information: the unrestricted offers case. *Operations Research*, 36:605–618, 1988.

[9] I. Cho. Uncertainty and delay in bargaining. *The Review of Economic Studies*, 57:575–595, 1990.

[10] O. Compte and P. Jehiel. The coalitional Nash bargaining solution. *Econometrica*, 78:1593–1623, 2010.

[11] B. Dutta and R. Vohra. Incomplete information, credibility and the core. *Mathematical Social Sciences*, 50:148–165, 2005.

[12] Y. Feinberg and A. Skrzypacz. Uncertainty about uncertainty and delay in bargaining. *Econometrica*, 73:69–91, 2005.

[13] F. Forges, J. F. Mertens, and R. Vohra. The ex ante incentive compatible core in the absence of wealth effects. *Econometrica*, 70:1865–1892, 2002.

[14] D. Fudenberg, D. Levine, and J. Tirole. Infinite-horizon models of bargaining with one-sided incomplete information. In A. Roth, editor, *Game Theoretic Models of Bargaining*. Cambridge University Press, 1985.

[15] D. Fudenberg and J. Tirole. *Game theory*. MIT press, 1991.

[16] S. Grossman and M. Perry. Sequential bargaining under asymmetric information. *Journal of Economic Theory*, 39:120–154, 1986.

[17] F. Gul. Bargaining foundations of Shapley value. *Econometrica*, 57:81–95, 1989.

[18] F. Gul and H. Sonnenschein. On delay in bargaining with one-sided uncertainty. *Econometrica*, 56:601–611, 1988.

[19] F. Gul, H. Sonnenschein, and R. Wilson. Foundations of dynamic monopoly and the Coase conjecture. *Journal of Economic Theory*, 39:155–190, 1986.

[20] B. Moldovanu and E. Winter. Order independent equilibria. *Games and Economic Behavior*, 9:21–34, 1995.

[21] R. Myerson. Mechanism design by an informed principal. *Econometrica*, 51:1767–1797, 1983.

[22] R. Myerson. Game theory: Analysis of conflict. *Harvard University Press*, 1991.

[23] A. Okada. A noncooperative coalitional bargaining game with random proposers. *Games and Economic Behavior*, 16:97–108, 1996.

[24] A. Okada. Non-cooperative bargaining and the incomplete informational core. *Journal of Economic Theory*, 147:1165–1190, 2012.

[25] A. Okada. A non-cooperative bargaining theory with incomplete information: verifiable types. *Journal of Economic Theory*, 163:318 – 341, 2016.

[26] M. Perry and P. Reny. A noncooperative view of coalition formation and the core. *Econometrica*, 62(4):795–817, 1994.

[27] D. Ray. *A game-theoretic perspective on coalition formation.* Oxford University Press, 2007.

[28] R. Selten. A noncooperative model of characteristic-function bargaining. In *Models of Strategic Rationality*, pages 247–267. Springer, 1988.

[29] R. Serrano and R. Vohra. Information transmission in coalitional voting games. *Journal of Economic Theory*, 134:117–137, 2007.

[30] A. Shaked. Opting out: bazaars versus 'hi tech' markets. *Investigaciones Economicas*, 18:421–432, 1994.